HW 1-12 page 1

2.2 p 119 55*, 57, 59*, 61, 63, 65*, 67, 77, 79

2.3 p 129 5*, 9, 11*, 13, 17, 21*, 23, 29, 33, 35, 41, 43, 47, 51, 55

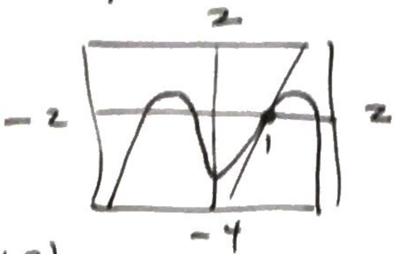
$\boxed{2.2}$ find the eq of the tangent line - calc to confirm

(55) $f(x) = -2x^4 + 5x^2 - 3$ @ $(1,0)$

$f'(x) = -8x^3 + 10x \Big|_{x=1} = 2$ ← slope

tang line $y - 0 = 2(x-1)$

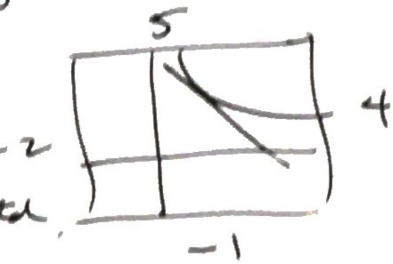
Confirm w/ calc.



(57) $f(x) = \frac{2}{x^{3/4}} = 2x^{-3/4}$ @ $(1,2)$

$f'(x) = -\frac{6}{4}x^{-7/4} \Big|_{x=1} = -\frac{3}{2}$ ← slope

tang. line $y - 2 = -\frac{3}{2}(x-1)$



find where there's a horizontal tangent line

(59) $y = x^4 - 2x^2 + 3$

$y' = 4x^3 - 4x = 4x(x^2-1)$

$y' = 0$ when $x = 0, \pm 1$

$(0,3), (-1,2), (1,2)$

(61) $y = \frac{1}{x^2} = x^{-2}$

$y' = -2x^{-3} = \frac{-2}{x^3}$ ← never zero slope

(63) $y = x + \sin x, \quad 0 \le x < 2\pi$

$y' = 1 + \cos x$ zero when $1 + \cos x = 0$

$\cos x = -1$

$x = \pi$

$(\pi, \pi + \sin\pi) = (\pi, \pi)$

2.2

65* Find k so f is tangent @ the line.

$f(x) = k - x^2$ line: $y = -6x + 1$

$f'(x) = -2x$ $m = -6$

$-2x = -6$ when $x = 3$

$k - x^2 = -6x + 1$

$k = x^2 - 6x + 1 \Big|_{x=3} = 9 - 18 + 1 = -8$

so k is -8

67. $f(x) = \frac{k}{x} = kx^{-1}$

line $= -\frac{3}{4}x + 3$ slope is $-\frac{3}{4}$ $f'(x) = -kx^{-2}$

where is same slope? $-kx^{-2} = -\frac{3}{4}$

$k = \frac{3x^2}{4}$

$f(x) = \frac{k}{x} = \frac{3x^2}{4x} = \frac{3x}{4}$ for all x so $(x, \frac{3}{4}x)$

What x has $\frac{3x}{4} = -\frac{3}{4}x + 3$

$(\frac{3}{4} + \frac{3}{4})x = 3$

$x = 3\left(\frac{4}{6}\right) = 2$

$f(2) = \frac{k}{2} = \frac{3(2)^2}{4} = 3$

so $k = 3$

(Check if $f(x) = \frac{3}{x}$ $f'(x) = \frac{-3}{x^2} \Big|_{x=2} = \frac{-3}{4}$ ✓)

$f(2) = \frac{3}{2}$

$y = \frac{-3}{4}(2) + 3 = \frac{3}{2}$ ✓

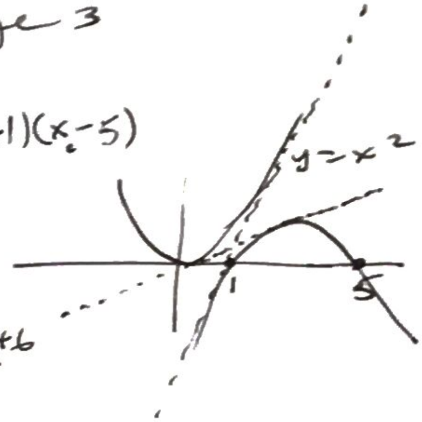
(77)

Sketch $y_1 = x_1^2$

$y_2 = -x_1^2 + 6x_1 - 5 = (-x+1)(x_2 - 5)$

Sketch 2 common tangents:

Find their equations:



$y_1' = 2x_1$ and $y_2' = -2x_2 + 6$

when the same?

$2x_1 = -2x_2 + 6 \longrightarrow$ so $x_1 = 3 - x_2$

Also $m = \dfrac{(-x_2^2 + 6x_2 - 5) - (x_1)^2}{x_2 - x_1} = \dfrac{-x_2^2 + 6x_2 - 5 - (3 - x_2)^2}{x_2 - (3 - x_2)}$

$= \dfrac{-x_2^2 + 6x_2 - 5 - (9 - 6x_2 + x_2^2)}{2x_2 - 3}$

$= \dfrac{-2x^2 + 12x_2 - 14}{2x_2 - 3} = y_2'$

So $-2x_2^2 + 12x_2 - 14 = y_2'(2x_2 - 3)$

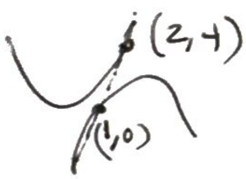
$-2x_2^2 + 12x_2 - 14 = (-2x_2 + 6)(2x_2 - 3)$

$-2x_2^2 + 12x_2 - 14 = -4x_2^2 + 18x_2 - 18$

$2x_2^2 - 6x_2 + 4 = 0$

$2(x_2 - 2)(x_2 - 1) = 0$, $x_2 = 1$ or 2

if $x_2 = 1$ then $x_1 = 3 - 1 = 2$



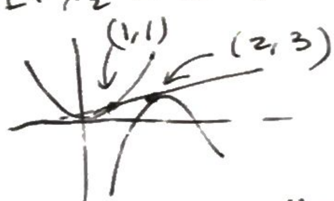
$m = \dfrac{4}{1}$

$y - 0 = 4(x - 1)$

$y = 4x - 4$

Check w/ graph Calc or GeoAlgebra

If $x_2 = 2$ then $x_1 = 3 - 2 = 1$



$m = \dfrac{2}{1}$

$y - 1 = 2(x - 1)$

$y = 2x - 1$

(79) Show $f(x) = 3x + \sin x + 2$ has no tangent line

$f'(x) = 3 + \cos x$

when is $f'(x) = 0$? when is $\cos x = -3$?

Never

HW 1-12 page 4 (Product / Quot Rule)

2.3 p 129 5, 9, 11, 13, 17, 21, 23, 29, 33, 35, 41, 43, 47, 51, 55

(5) $g(x) = (2x-3)(1-5x)$

$g'(x) = (2x-3)(-5) + (2)(1-5x)$

$= -10x + 15 + 2 - 10x = -20x + 17$

(9) $f(x) = x^3 \cos x$

$f'(x) = x^3(-\sin x) + 3x^2 \cos x$

(11) $f(x) = \frac{x}{x-5}$

$f'(x) = \frac{(x-5)(1) - x(1)}{(x-5)^2} = \frac{-5}{(x-5)^2}$

(13) $h(x) = \frac{\sqrt{x}}{x^3+1}$

$h'(x) = \frac{(x^3+1)(\frac{1}{2}) - \sqrt{x}(3x^2)}{(x^3+1)^2}$

(17) $f(x) = (x^3+4x)(3x^2+2x-5)$

$f'(x) = (x^3+4x)(6x+2) + (3x^2+4)(3x^2+2x-5)$

$f'(x) = 6x^4 + 2x^3 + 24x^2 + 8x + 9x^4 + 6x^3 - 15x^2 + 12x^2 + 8x - 20$

$f'(x) = 15x^4 + 8x^3 + 21x^2 + 16x - 20$

$f'(0) = -20$

(21) $f(x) = x \cos x$

$f'(x) = x(-\sin x) + (1)(\cos x)$

$f'(x) = -x \sin x + \cos x$

$f'(\frac{\pi}{4}) = \frac{-\pi}{4} \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \frac{-\sqrt{2}}{8} + \frac{\sqrt{2}}{2}$

(23) $y = \frac{x^3+6x}{3} = \frac{1}{3}x^3 + 2x$

$y' = x^2 + 2$

(25) $y = \frac{6}{7x^2} = \frac{6}{7}x^{-2}$

$y' = \frac{6}{7}(-2x^{-3}) = \frac{-12}{7x^3}$

(27) $y = \frac{4x^{3/2}}{x} = 4x^{1/2}$ $y' = 2x^{-1/2} = \frac{2}{\sqrt{x}}$

HW 1-12 page 5

(29) $f(x) = \dfrac{4-3x-x^2}{x^2-1}$ so $f'(x) = \dfrac{(x^2-1)(-3-2x)-(4-3x-x^2)(2x)}{(x^2-1)^2}$

(33) $f(x) = \dfrac{3x-1}{\sqrt{x}}$ so $f'(x) = \dfrac{\sqrt{x}(3)-(3x-1)(\frac{1}{2}x^{-1/2})}{x}$

(or $\frac{d}{dx}[3x^{1/2}-x^{-1/2}]$
$= \frac{3}{2}x^{-1/2}+\frac{1}{2}x^{-3/2}$)

(37) $g(s) = s^3\left(5-\dfrac{s}{s+2}\right)$

$g(s) = 5s^3 - \dfrac{s^4}{s+2}$

$g'(s) = 15s^2 - \dfrac{(s+2)(4s^3)-s^4(1)}{(s+2)^2}$

(41) $f(t) = t^2 \sin t = t^2(\cos t) + (2t)(\sin t)$

(45) $g(t) = -x + \tan x \qquad g'(t) = -1 + \sec^2 x$

(51) $y = -\csc x - \sin x$

$y = \dfrac{-1}{\sin x} - \sin x$

$y' = \dfrac{\sin x(0)-(-1)(\cos x)}{\sin^2 x} - \cos x$

$y' = \dfrac{\cos x}{\sin^2 x} - \cos x$ or $\dfrac{\cos x(1-\sin^2 x)}{\sin^2 x}$

$y' = \cot x \csc x - \cos x$ or $\dfrac{\cos^3 x}{\sin x}$

or ..... many trig identities here! or $\cos x \cot x$ etc....